

Holographic dark energy in the DGP model

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(Dated: September 12, 2011)

The braneworld model proposed by Dvali, Gabadadze and Porrati leads to an accelerated universe without cosmological constant or other form of dark energy. Nevertheless, we have investigated the consequences of this model when an holographic dark energy is included, taken the Hubble scale as IR cutoff. We have found that the holographic dark energy leads to an accelerated universe flat (de Sitter like expansion) for the two branch: $\epsilon = \pm 1$ of the DGP model. Nevertheless, in universes with no null curvature the dark energy presents an EoS corresponding to a phantom fluid during the present era and evolving to a de Sitter like phase for future cosmic time. In the special case in which the holographic parameter c is equal to one we have found a sudden singularity in closed universes. In this case the expansion is decelerating.

I. INTRODUCTION

The acceleration in the expansion of the universe during recent cosmological times, first indicated by Supernovae observations [1] and also supported by the astrophysical data obtained from WMAP, indicates, in the framework of general relativity, the existence of an exotic fluid with negative pressure, a form of matter which received the rather confusing name of dark energy. Other non conventional approaches have advocated extra dimensions inspired in string and superstring theories. One of this models that have leads to an accelerated universe without cosmological constant or other form of dark energy is the braneworld model proposed by Dvali, Gabadadze, and Porrati (DGP) [2], [3], [4] (for reviews, see [5]). In a cosmological scenario, this approach leads to a late-time acceleration as a result of the gravitational leakage from a 3-dimensional surface (3-brane) to a 5-th extra dimension on Hubble distances.

More specifically, this model leads to an accelerated phase at late times but with an effective dark energy component with $w > -1$, and dependent of the redshift. Since observations do not exclude the possibility of crossing the phantom divide with an effective energy density with an EoS with $w < -1$, recently Hirano and Komiya [6] have extended the modified Friedmann equation proposed by Dvali and Turner [7], in order to realize this crossing.

It is a well known fact that the DGP model has two branches of solutions: the self-accelerating branch and the normal one. The self accelerating branch leads to an accelerating universe without invoking any exotic fluid, but present problems like ghost [8]. Nevertheless, the normal branch requires a dark energy component to accommodate the current observations [9], [10]. Extend models of gravity on the brane with $f(R)$ terms have been investigated to obtain self acceleration in the normal branch [11]. Solutions for a DGP brane-world cosmology with a k-essence field were found in [12] showing big rip scenarios and asymptotically de Sitter phase in the future.

The aim of the present work was to explore, in the framework of the holographic dark energy models [13], [14], [15], based on the holographic principle [16], which is believed

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to be a fundamental principle for the quantum theory of gravity, a DGP cosmology. We investigate the behavior of a late time universe, considering dark energy as unique dominant fluid.

Based on the validity of the effective quantum field theory, Cohen et al [13] suggested that the total energy in a region of size L should not exceed the mass of a black hole of the same size, which means $\rho_\Lambda \leq L^{-2}M_p^2$. The largest L is chosen by saturating the this bound so that we obtain the holographic dark energy (HDE) density

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \quad (1)$$

where c is a free dimensionless $\mathcal{O}(1)$ parameter that can be determined by observations. Taking L as the Hubble radius $H = H_0^{-1}$ this ρ_Λ is comparable to the observed dark energy density, but gives wrong EoS for the dark energy [14].

For higher dimensional space-times, the holographic principle in cosmological scenarios has been formulated considering the maximal uncompactified space of the model, i.e. in the bulk, leading to a crossing of phantom divide for the holographic dark energy, in 5D two-brane models [17]. Recently, a modified holographic dark energy model has been formulated using the mass of black holes in higher dimensions and the Hubble scale as IR cutoff [18]. Using the future event horizon as IR cutoff, it was found in that the EoS of the holographic dark energy can cross phantom divide [19].

We explore in this work the usual holographic bound in a DGP model with curvature. We find that using the Hubble scale as IR cutoff, the universe with the observable values corresponding to curvature, the EoS of the dark energy can cross the phantom divide and end in a de Sitter like expansion.

In the next section we present the DGP model for a universe with curvature but without matter. In section III, the EoS of the holographic dark energy with a Hubble cutoff is evaluated for universes with no null curvature. In section IV we discuss our results.

II. DGP MODEL

For an homogeneous and isotropic universe described by the Friedmann-Robertson-Walker the field equation is given by [3], [4] (with $8\pi G = 1$)

$$3 \left(H^2 - \frac{\epsilon}{r_c} \sqrt{H^2 + \frac{k}{a^2}} \right) = \rho - \frac{3k}{a^2}, \quad (2)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ is the total cosmic fluid energy density on the brane, and r_c , given by

$$r_c = \frac{1}{2} \frac{M_{(4)}^2}{M_{(5)}^3}, \quad (3)$$

is a scale which sets a length beyond which gravity starts to leak out into the bulk. The parameter $\epsilon = \pm 1$ represents the two branches of the DGP model. It is well known that the solution with $\epsilon = +1$ represent the self-accelerating branch, since even without dark energy the expansion of the universe accelerates, and for late times the Hubble parameter approaches a constant, $H = 1/r_c$. In the previous investigation, $\epsilon = -1$ has been named as the normal branch, where acceleration only appears if a dark energy component is included.

Before to discuss the holographic behavior of dark energy in a DGP model where no null curvature is taken account, we present briefly the solutions without dark energy ($\rho = 0$). For this case Eq.(2) becomes

$$H^2 + \frac{k}{a^2} = \frac{\epsilon}{r_c} \sqrt{H^2 + \frac{k}{a^2}}, \quad (4)$$

so only the cases $k = \pm 1$ and $\epsilon = 1$ admit physically reasonable solutions. The equation to solve is then

$$H^2 + \frac{k}{a^2} = \frac{1}{r_c^2}. \quad (5)$$

which gives us the following solutions

$$a(t) = \begin{cases} r_c \sinh\left(\frac{t}{r_c} + \text{const.}\right), & k = -1, \\ r_c \cosh\left(\frac{t}{r_c} + \text{const.}\right), & k = 1. \end{cases} \quad (6)$$

The case $k = 0$ correspond a de Sitter universe with $H = 1/r_c$.

Let us consider a general behaviors of the model for a flat universe when one fluid is included, in terms of the size of the quantity $r_c H$. Rewriting Eq. (2) using $r_c H$ yields

$$3H^2 [1 - \epsilon (r_c H)^{-1}] = \rho. \quad (7)$$

If cosmic fluid satisfy a barotropic equation of state, $p = \omega \rho$, the conservation equation is given by

$$\dot{\rho} + 3H(1 + \omega)\rho = 0. \quad (8)$$

From Eqs. (7) and (8), we obtain an expression for ω in terms of the Hubble parameter, which is given by

$$1 + \omega = -\frac{1}{3} \left(\frac{2 - \epsilon(r_c H)^{-1}}{1 - \epsilon(r_c H)^{-1}} \right) \frac{\dot{H}}{H^2}. \quad (9)$$

According to Eq. (9), $1 + \omega < 0$ implies that $\dot{H} > 0$, since $(2 - \epsilon(r_c H)^{-1}) / (1 - \epsilon(r_c H)^{-1}) > 0$, for both cases $(r_c H)^{-1} \lesseqgtr 1$.

Using expressions (7) and (9), we find special limits in terms of the size of $(r_c H)^{-1}$. Let see the three following cases: a) If $(r_c H)^{-1} \ll 1$, then $3H^2 \rightarrow \rho$ and $1 + \omega \rightarrow -\frac{2}{3} \frac{\dot{H}}{H^2}$, which correspond to the standard cosmology, in a four dimensional spacetime. b) If $(r_c H)^{-1} \gg 1$, then $3H \rightarrow r_c \rho$, where we have assumed $\epsilon = -1$, so for a positive energy density, the Hubble parameter is also positive. Independently of the value of ϵ , we obtain for the equation of state, $1 + \omega \rightarrow -\frac{1}{3} \frac{\dot{H}}{H^2}$.

III. THE HOLOGRAPHIC DARK ENERGY

A. Hubble cut-off

In what follows we shall consider an holographic dark energy, which obeys the relation

$$\rho = 3c^2 H^2. \quad (10)$$

Case $k = 0$. Using Eq. (10) in Eq. (7), a direct solution for the Hubble parameter is obtained

$$H = \frac{\epsilon}{(1 - c^2) r_c}, \quad (11)$$

along with an equation of state $\omega = -1$. In this scenario the universe accelerates with a constant Hubble parameter, which differs only by a factor $\epsilon/(1 - c^2)$ compared with the self accelerating case, which appears in the branch $\epsilon = 1$, without dark energy. Note that the value of the holographic parameter c allows the following two cases, both with acceleration: i) $\epsilon = 1$ and $c^2 < 1$, ii) $\epsilon = -1$ and $c^2 > 1$. The cases with $c \lesseqgtr 1$ were well discussed from theoretical point of view in [20], [21] and from the observational point of view in [22]. Then, in principle, both branch can behaves as a de Sitter universe.

In order to have appreciable modifications from the standard cosmology at late times of cosmic evolution, it has been usually assumed that $r_c \sim H_0^{-1}$ in the DGP model. So in

the holographic approach with the Hubble cutoff we obtain both possible cases implies: i) $\epsilon = 1$ and $c^2 \sim 0$, ii) $\epsilon = -1$ and $c^2 \sim 2$. From Eq.(10), the first case indicates that the holographic dark energy density is approximately zero. In other words, holography leads to obtain the previous case of de Sitter universe without matter in the flat case. In the second one, holography implies a de Sitter universe with a high density of dark energy. In this sense, the inclusion of holography in the normal branch $\epsilon = -1$ leads to an scenario with accelerated expansion but with mayor stuff of dark energy.

Cases with no null curvature. For universes with non null curvature, Eq.(2) can be solved for the Hubble parameter in terms of the variable $x = r_c/a$.

$$[(1 - c^2) r_c H_{\pm}(x)]^2 = \frac{1}{2} \left[1 - \delta(x) \pm \sqrt{\Delta(x)} \right], \quad (12)$$

where $\delta(x) = 2k(1 - c^2)x^2$ and $\Delta(x) = 1 - 4kc^2(1 - c^2)x^2$. In order to recover the flat case, the physical solution will be $H_+(x)$. Imposing the constraint $\Delta(x) > 0$ we obtain that $0 \leq x^2 < 1/4c^2(1 - c^2)$, for $k = 1$ and $c^2 < 1$, and $0 \leq x^2 < 1/4c^2(c^2 - 1)$, for $k = -1$ and $c^2 > 1$. Note that these conditions implies $\delta(x) > 0$ and consequently $1 - \delta(x) + \sqrt{\Delta(x)} > 0$.

The constraints upon x implies the that the scale factor a has the following lower bounds $a > 2cr_c\sqrt{1 - c^2}$, or, $a > 2cr_c\sqrt{c^2 - 1}$. In terms of the redshift these bounds are $1 + z < a_0/2cr_c\sqrt{1 - c^2}$, or, $1 + z < a_0/2cr_c\sqrt{c^2 - 1}$

From Eq.(8), and since $1 + \omega(x) = (2/3)(x/H) dH/dx$, and using Eq.(16), we obtain the following expression for the equation of state

$$1 + \omega(x) = -\frac{4}{3}k(1 - c^2) \left(\frac{1 + c^2/2\sqrt{\Delta(x)}}{1 - \delta(x) + \sqrt{\Delta(x)}} \right) x^2 \quad (13)$$

Since the factor $k(1 - c^2) > 0$, due to the conditions mentioned above, Eq.(13) tell us that we have phantom evolution for $x > 0$ (or $a > 0$). Nevertheless, a de Sitter phase is achieved in the final evolution when $x = 0$ (or $a = \infty$).

In order to compare our model with some observed values for the present EoS of the dark energy, we will see the behavior of EoS, given by Eq.(13), when $r_c \sim H_0^{-1}$. Parameterizing the above condition, we can write $r_c H_0 = \alpha$, where $\alpha \sim 1$. Redefining the variable kx^2 as $\eta(z)$, yields

$$\eta(z) = \alpha^2 \Omega_k(0)(1 + z)^2, \quad (14)$$

where $\Omega_k(0)$ is the density parameter of the curvature at the present time. Then in terms of the redshift Eq.(13) becomes

$$1 + \omega(z) = -\frac{4}{3}(1 - c^2) \left(\frac{\left(1 + c^2/2\sqrt{1 - 4c^2(1 - c^2)\eta(z)}\right)}{1 - 2c^2(1 - c^2)\eta(z) + \sqrt{1 - 4c^2(1 - c^2)\eta(z)}} \right) \quad (15)$$

The value of the holographic parameter c^2 has been fitted with the current data for a variety of models and IR cutoff (See [23]). Since in our case we are interested in the very late time behavior we will use values for c^2 imposed above for a closed and open universes. Using the last expression we have plotted $\omega(z)$ in Fig. 1, for three values of the parameter α . We have considered the case of a closed universe and take the value $\Omega_k(0) = 0.0049$, which is within the range given by WMAP observations (See [24]). The EoS of the holographic dark energy behaves like a phantom fluid during the present time of the universe, but practically indistinguishable of a cosmological constant. Fig. 2 shows a similar behavior when α is fixed and the holographic parameter is varied. Note, nevertheless, that for an open universe the EoS correspond to a phantom fluid, with values within the observational data, in the both cases with c^2 fixed (Fig. 3) and with α fixed (Fig.4).

The special case $c = 1$. In this case, the equation for H becomes

$$\frac{\epsilon}{r_c} \sqrt{H^2 + \frac{k}{a^2}} = \frac{k}{a^2}, \quad (16)$$

which has the following solution for the scale factor if the initial condition is $a(t = t_0) = a_0$

$$ka^2(t) = r_c^2 - (t_s - t)^2, \quad (17)$$

where we have chosen $\epsilon = 1$. For a closed universe a sudden singularity occurs at the time t_s , which is given by

$$t_s = t_0 + \sqrt{r_c^2 - ka_0^2}. \quad (18)$$

Using Eq.(17) we can evaluate the EoS from the expression $1 + \omega = -2\dot{H}/H^2$

$$1 + \omega = \frac{2}{3} \left(1 + \frac{r_c^2}{(t_s - t)^2} \right). \quad (19)$$

The above equation indicates that the holographic dark energy has an EoS with $\omega > -1/3$, indicating a non accelerated universe, and a sudden singularity at $t = t_s$, since $1 + \omega = \infty$ and the scale factor is finite: $a^2(t = t_s) = r_c^2$. Additionally, the Hubble parameter, given by

$$H^2(t) = \frac{1}{r_c^2 - (t_s - t)^2} \left(\frac{r_c^2}{r_c^2 - (t_s - t)^2} - 1 \right), \quad (20)$$

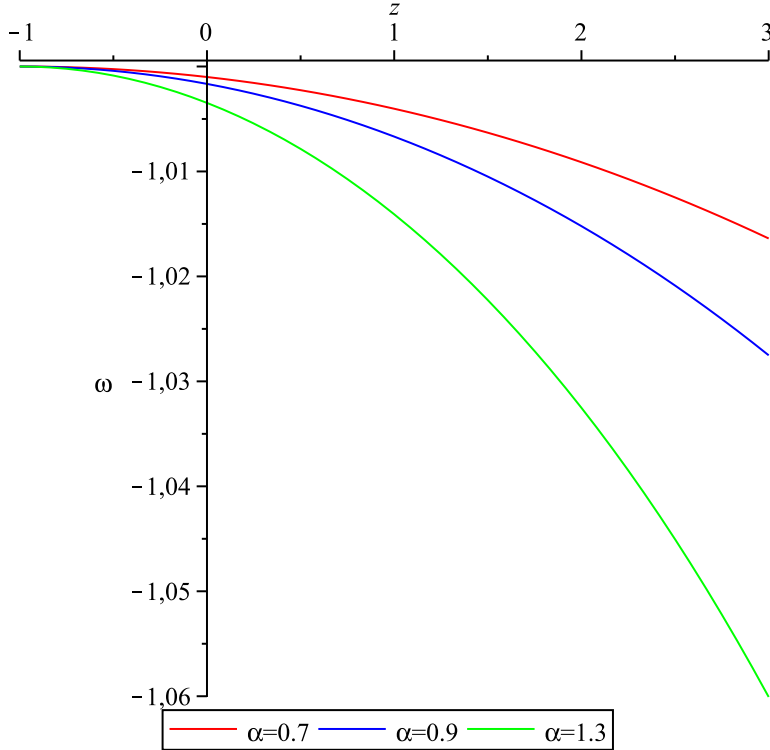


FIG. 1: EoS as a function of the redshift, for a closed universe, for three values of α and $c^2 = 0.25$.

is zero at $t = t_s$ along with the dark energy density. The pressure is then finite at this time. Finally, the acceleration at $t = t_s$ is $\ddot{a}/a = -1/r_c^2$.

IV. CONCLUSIONS

It is a well known fact that the DGP models do not require a dark energy to obtain an accelerated universe. Nevertheless, we have investigated the behavior of the cosmic evolution when a holographic dark energy is included, using the Hubble scale as cutoff. We have considered the curvature in general, obtaining non trivial evolutions when the universes are closed or open.

Since the condition $r_c \sim H_0^{-1}$ is imposed in the DGP framework to obtain only appreciable modifications to the standard cosmology at late times, we have considered our results under this condition.

For a flat universe, the holographic dark energy leads to an accelerated universe (de

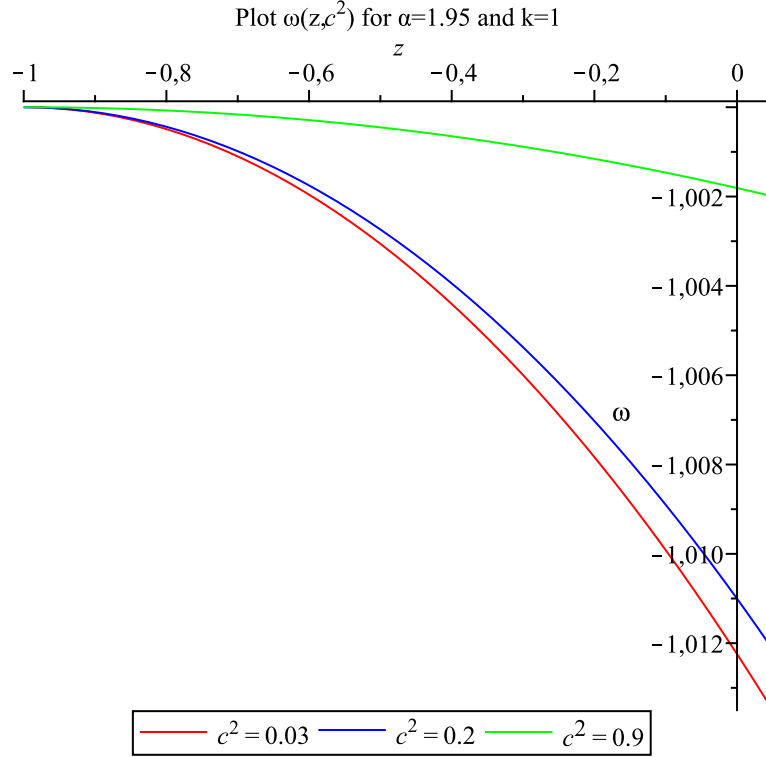


FIG. 2: EoS as a function of the redshift, for a closed universe, for three values of c^2 $\alpha = 1.95$.

Sitter like expansion) for the two branch: $\epsilon = \pm 1$, under the following conditions for the holographic parameter c : i) $\epsilon = 1$ and $c^2 < 1$, ii) $\epsilon = -1$ and $c^2 > 1$. In this scenario the universe accelerates with a constant Hubble parameter, which differs only by a factor $\epsilon/(1 - c^2)$ compared with the self accelerating case, which appears in the branch $\epsilon = 1$, without dark energy. Imposing the condition $r_c \sim H_0^{-1}$ we obtain that if we are in the branch $\epsilon = 1$ the holographic dark energy density is approximately zero, which implies that holography leads to obtain the previous case of de Sitter universe without matter in the flat case. In the branch $\epsilon = -1$, usually named as the normal branch, we obtain a de Sitter universe with a high density of dark energy, which is also similar to the results previous obtained where acceleration only appears if a dark energy component is included.

In the cases with no null curvature we obtain the following restrictions for the holographic parameter: i) if $k = 1$, then $c^2 < 1$, and ii) if $k = -1$, then $c^2 > 1$. The scale factor a of these models has the following lower bounds $1 + z < a_0/2cr_c\sqrt{1 - c^2}$, and $1 + z < a_0/2cr_c\sqrt{c^2 - 1}$. The main result of the holographic DGP models with curvature is that the dark energy

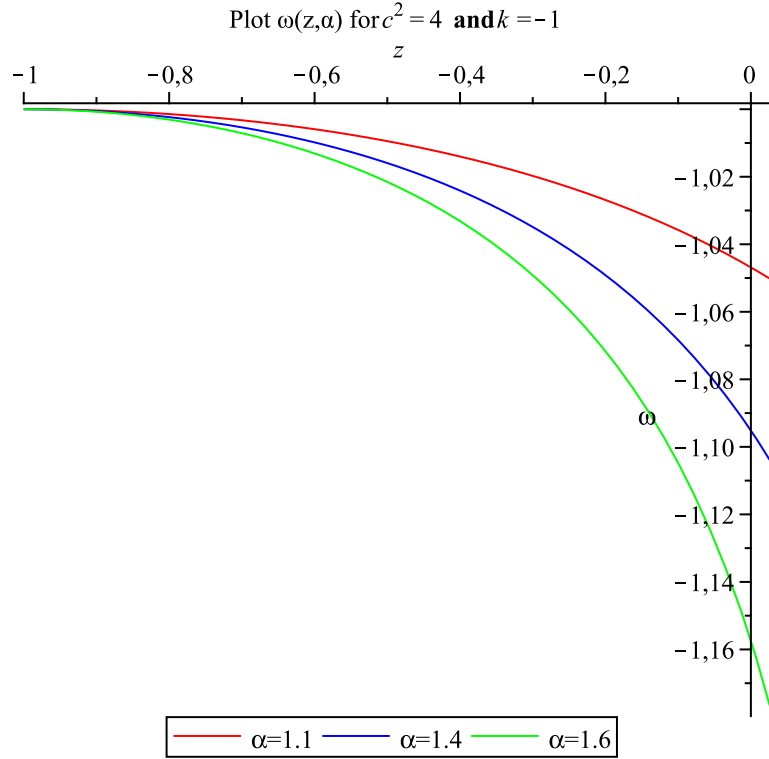


FIG. 3: EoS as a function of the redshift for an open universe, for three values of α and $c = 4.0$.

presents an EoS corresponding to a phantom fluid during the present era and evolving to a de Sitter like phase for future cosmic time. Our results indicate that the holographic parameter c has a little influence in the present values of the phantom EoS of the dark energy. Physically, it means that the increasing or decreasing of the holographic dark energy is not a relevant factor in our model. On the other hand, the condition $r_c \sim H_0^{-1}$, which was parameterized throughout the parameter α , measures the modifications that induces the DGP approach on the standard field equations at the present times. We have obtain that the phantom behavior increases if α increases, or if $r_c \lesssim H_0^{-1}$.

V. ACKNOWLEDGEMENTS

NC and SL acknowledge the hospitality of the Physics Department of Universidad de La Frontera where part of this work was done. SL and FP acknowledge the hospitality of the Physics Department of Universidad de Santiago de Chile. We acknowledge the support

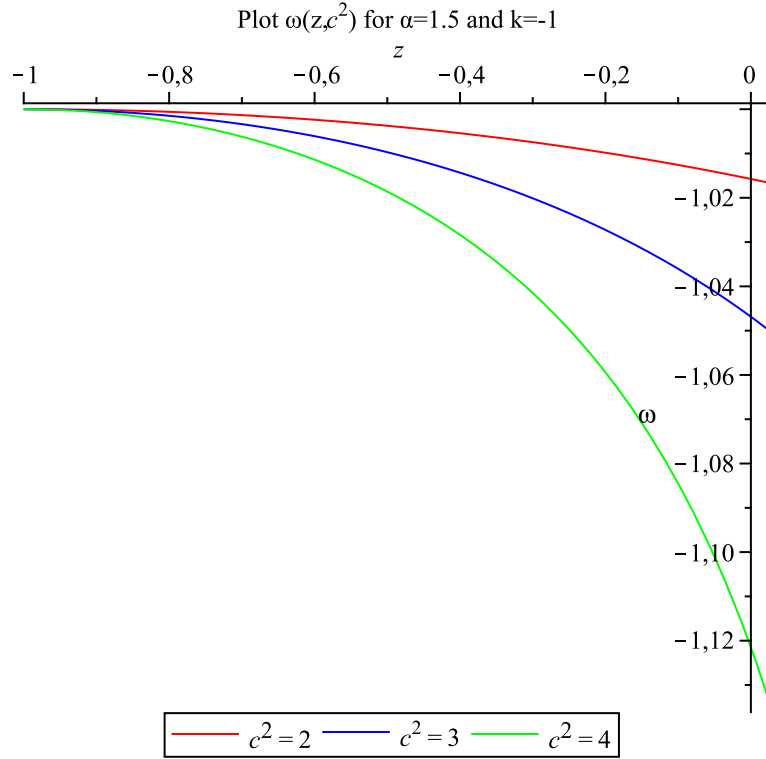


FIG. 4: EoS as a function of the redshift for an open universe, for three values of c^2 and fixed α .

to this research by CONICYT through grants Nos. 1110840 (NC) and 1110076 (SL). This work was also supported from DIUFRO DI10- 0009, of Dirección de Investigación y Desarrollo, Universidad de La Frontera (FP) and DIR01.11,037.334/2011, VRIEA, Pontificia Universidad Católica de Valparaíso (SL).

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